ACEEE Int. J. on Communication, Vol. 02, No. 03, Nov 2011

Design And Performance of Finite impulse Response Filter Using Hyperbolic Cosine Window

Harish Kumar,[1], Piyush Kumar[1] and Kaushal Kumar[1],Venkat babu G[2]
[1] Bhagwant Institute of Technology ,Muzaffarnagar India
Email: hc78@rediffmail.com
[2] Sharda University, School of Engineering, Greater Noida, India
Email:gvenkatbabu@rediffmail.com

***Abstract*—In this paper a proposed of design and analysis of Finite impulse response filter using Hyperbolic Cosine window (Cosh window for short). This window is very useful for some applications such as beam forming, filter design, and speech processing. Digital FIR filter designed by Kaiser window has a better far-end stop-band attenuation than filter designed by the other previously well known adjustable windows such as Dolph-Chebyshev and Saramaki, which are special cases of Ultraspherical windows, but obtaining a digital filter which performs higher far-end stop band attenuation than Kaiser window will be useful. In this paper, the design of nonrecursive digital FIR filter has been proposed by using Cosh window. It provides better side lobe roll-off ratio & far-end stop band attenuation than filter designed by well known Kaiser window, which is the advantage of filter designed by Cosh window over filter designed by Kaiser window. An expression for the side lobe & far field level has been developed. Simulation & experimental results showing a good agreement with theory has been provided**

***Index Terms*— Digital FIR filter, far-end stopband attenuation, side-lobe roll-off ratio, window technique, Kaiser Window, Hyperbolic Cosine window.**

I. Introduction

The Window functions are widely used for different applications of Digital Signal processing such as signal analysis, signal estimation, Digital filter design and speech processing [1]-[2]. Number of windows has been proposed for different applications [3]-[6]. They are having the suboptimal solutions and depending upon the applications. In the literature, the Kaiser window is known as flexible window and this is widely used for the applications of Digital filter design and spectrum analysis [2]-[3]. This is because it achieves close approximation to the discrete prelate spheroid functions that have maximum energy concentration in the main lobe. Window length N and shape parameter á are two independent parameters of Kaiser Window and by adjusting these parameters, the spectral parameters such as main lobe width, ripple ratio and side-lobe roll off ratio for different applications can be controlled.

The side-lobe roll off ratio is important parameter for some applications such as beam forming [6], digital filter design and speech processing [7]. For beam forming applications, the large side-lobe roll off ratio is required to reject far end interference better [6]. For filter design applications, it can reduce the far end attenuation for stop band energy and for speech processing, it reduces the energy leak from one band to another [7].

Among all the well-known adjustable windows such as Dolph-Chebyshev [4] and Saramaki [5] which are special cases of Ultra spherical window [6], The Kaiser window has a better side-lobe roll off characteristics. But obtaining a window which provides higher sidelobe roll off characteristics than Kaiser Window will be useful for some applications.

The paper is organized as follow; the spectral parameters of window are described in section II. The proposed window is explained in section III. The application of proposed window to design nonrecursive FIR filter is explored in section IV. The simulation results are given in section V and this paper is concluded in section VI. The paper is also equipped with the references given at the end of this paper.

II. Spectral Parameters of Window

A window w (nT) with length of N is a Time-Domain function and it is nonzero for $n \le |(N-1)/2|$ and zero for otherwise. The frequency spectrum of w(nT) can be found as

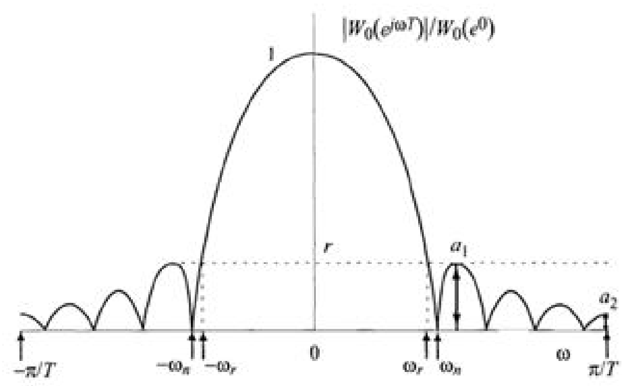

Figure 1. Amplitude spectrum and some common spectral characteristics of a typical normalized window [10]

$$W(e^{jwT}) = e^{-j\theta(w)}\, W_0\,(e^{jwT}) \quad (1)$$

Where $W_o(e^{jwT})$ is called the amplitude function. The amplitude and phase spectrums of a window are given by $A(w) = |Wo(e^{jwT})|$ and $\theta(w)$, respectively, and $|Wo(e^{jwT})| / Wo(e^{o})$ is a normalized version of the amplitude spectrum. A typical window's normalized amplitude spectrum and some common spectral characteristics are depicted in Figure 1. Important window parameter is the ripple ratio *r* which is defined as

DOI: 01.IJCOM.02.03. 6

$$r = \frac{\text{maximum sidelobe amplitude}}{\text{main lobe amplitude}} \quad (2)$$

The ripple ratio is a small quantity less than unity and, in consequence, it is convenient to work with the reciprocal of *r* in dB, i.e.

$$R = 20\log_{10}(1/r) \quad (3)$$

where R can be interpreted as the minimum side-lobe attenuation (minimum stopband attenuation) relative to the main lobe and *-R* is the ripple ratio in dB. Another parameter used to describe the side-lobe pattern of a window is the side-lobe roll-off ratio, *s*, which is defined as

$$S = (a_1/a_2) \quad (4)$$

Where, a_1 and a_2 are the amplitudes of the side lobe nearest and furthest, respectively, from the main lobe (see Figure 1). If S is the side-lobe roll-off ratio in dB, then s is given by

$$S = 10^{s/20} \quad (5)$$

For the side-lobe roll-off ratio to have meaning, the envelope of the side-lobe pattern should be monotonically increasing or decreasing. The side-lobe roll-off ratio provides a description of the distribution of energy throughout the side lobes, which can be of importance if prior knowledge of the location of an interfering signal is known. Further explanation of the usefulness of these spectral characteristics can be found in [11].

III. Proposed Window

It can be seen that the functions *cosh(x)* and $I_o(x)$ (Zero order Bessel function of first kind used for Kaiser window) have the same shape characteristics which is exponential in nature that is shown in figure 2, Therefore, a new window, namely Cosh window, can be proposed as

$$(k) = \frac{cosh(\alpha_{ch}\sqrt{1-(\frac{2n}{N-1})^2})}{cosh(\alpha_{ch})} \quad , \quad |n| \le \frac{N-1}{2} \quad (6)$$

The normalized spectrum of proposed window (in dB) can be obtained by

$$W_N(e^{jwT}) = 20\,log_{10}(\frac{|A(w)|}{|A(w)|_{max}}) \quad (7)$$

Figure 3 shows the frequency spectrum of proposed window for a fixed value of filter length N = 51. The parameter α_{ch} =0 corresponds to the rectangular window. From figure 3, it can be easily seen that, when α_{ch} increases then the main lobe width increases and ripple ratio decreases.

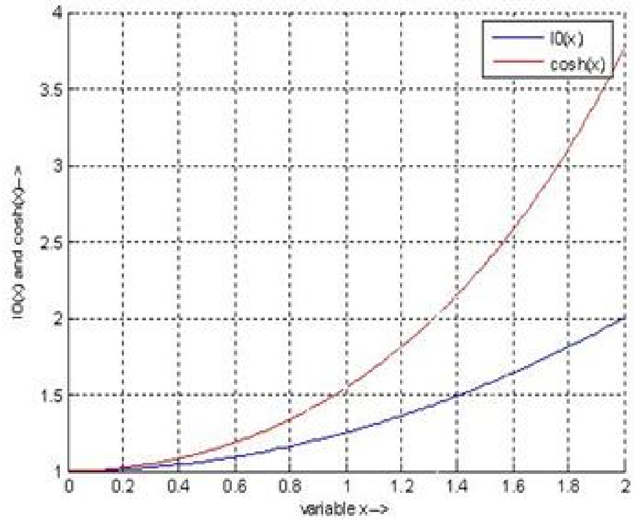

Figure 2. Similar shape characteristics of function cosh(x) and $I_o(x)$

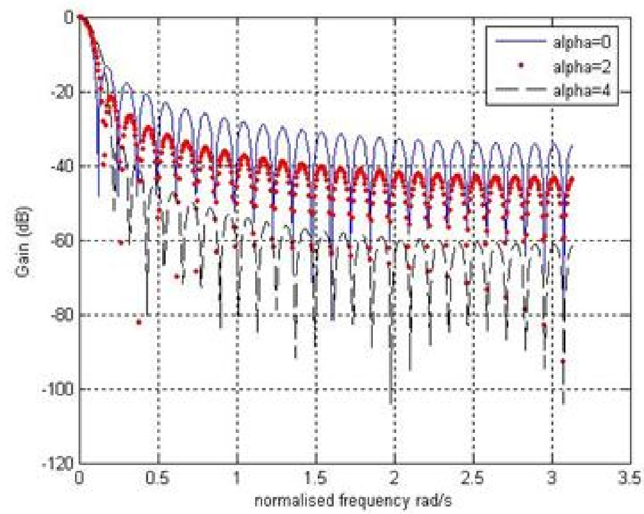

Figure 3. Proposed window spectrum in dB for α_{ch}= 0, 2, and 4 and N=51

IV. Application To Design FIR Filter

A. Filter Design using Cosh window

Fourier series method with windowing is the most straightforward technique to design FIR filters and involves a minimal amount of computation compared to the optimization methods. The aim to use a window in Fourier series method is to truncate and smooth the infinite duration impulse response of the ideal prototype filter. The impulse response of a realizable noncasual FIR filter using a window function, w(nT), is obtained as

$$h_{nc}(nT) = w(nT)h_{id}(nT) \quad (8)$$

where $h_{id}(nT)$ is the infinite duration impulse response of the ideal filter. For a low pass filter with a cut off frequency, w_c, and sampling frequency, w_s, it can be found as [1]

$$h_{id}(nT) = \begin{cases} \frac{w_cT}{\pi} & ; n = 0 \\ \frac{sinw_cnT}{n\pi} & ; w_c \le |w| \le w_s/2 \end{cases} \quad (9)$$

DOI: 01.IJCOM.02.03.6

Delaying the non casual impulse response $h_{nc}(nT)$ by a period (N-1)/2, a casual filter can be obtained as

$$h(nT) = h_{nc}\left[\left(n - \frac{N-1}{2}\right)T\right]; 0 \leq n \leq N-1 \qquad (10)$$

The ripples in both passband and stopband regions of the filters designed by the window method are approximately equal to each other [3].

The frequency response of digital FIR filter designed by Cosh window is shown in figure 4 which shows the effect of parameter (α_{ch}) on the filter characteristic. It can be seen from figure 4, an increase in α_{ch} results in better minimum stopband attenuation (A_s) but a worse transition width.

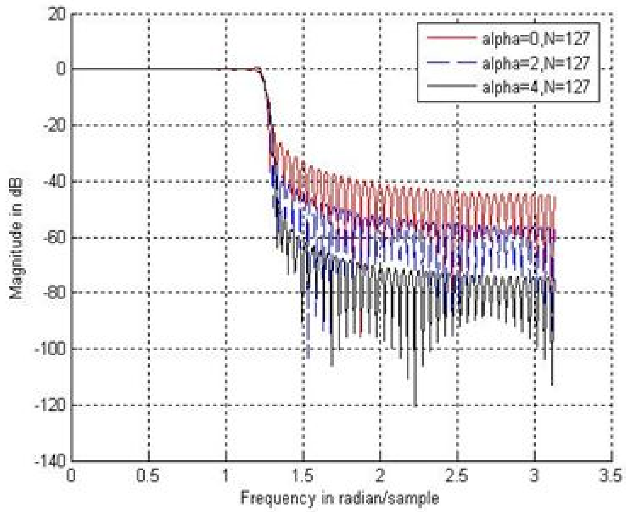

Figure 4. Amplitude spectrums of the filters designed by the Cosh window for various α_{ex} with N=127

B. *Filter Design Equation for Proposed Window*

To find the suitable window which satisfies the given prescribed filter specification, we obtain the relation between the window parameters and filter parameters. Figure 5 shows the relation between Cosh window parameter, α_{ch}, and the minimum stopband attenuation, A_s, for N=127. From figure, it is clear that as the window parameter increases, the minimum stopband attenuation A_s also increases. By using the quadratic polynomial curve fitting method, the first design equation is obtained as

$$\alpha_{ch} = \begin{cases} -0.0003251A_s^2 + 0.1677A_s - 3.149; & A_s \leq 20.77 \\ 0 & ; A_s > 20.77 \end{cases} \qquad (11)$$

The second filter design equation is the relation between minimum stopband attenuation A_s and normalized width, D, which is required to find the minimum length of the filter [6]. The normalized width parameter can be calculated by the following equation [6].

$$D = \frac{\Delta w(N-1)}{w_s} \qquad (12)$$

where Δw is the transition bandwidth. The relation between D and A_s is shown in figure 6. The comparison between the filters designed by Kaiser window and proposed window is also shown in Figure 6. Figure 6 indicates that the filters designed by Kaiser window have better minimum stopband attenuation characteristic than the filters designed by exponential window. By using quadratic curve fitting method, an approximate expression for D can be found as D =
$\{2.692 \times 10^{-5} A_s^2 + 0.06959 A_s - 0.2385; A_s > 20.77\}$
$\{0 \qquad ;A_s$ d"20.77$\}$ (13)
By using (12) and (13), the minimum odd integer filter length required for satisfying a given A_s and Δw can be determined from

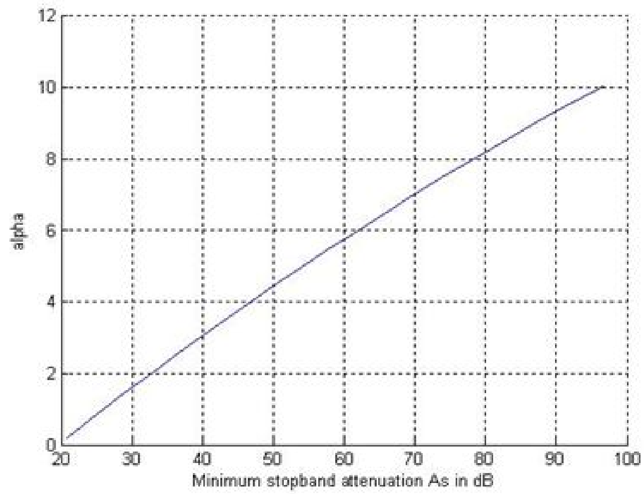

Figure 5. shows the relation between Cosh window

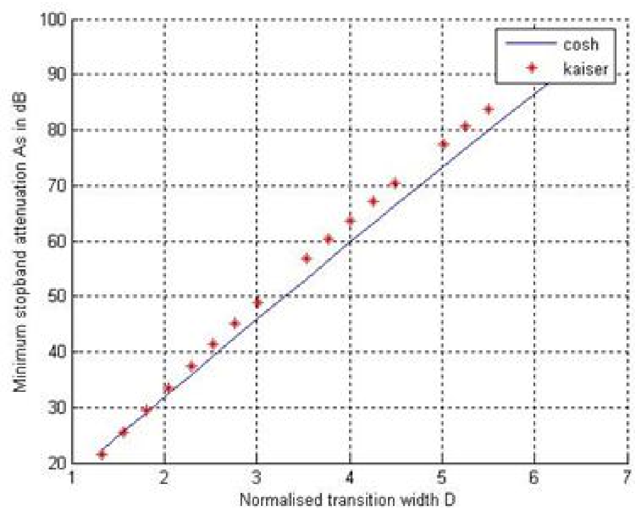

Figure 6. Comparison of the filters designed by Cosh and Kaiser windows in terms of the minimum stopband attenuation with N=127

$$N = \frac{Dw_s}{\Delta w} + 1 \qquad (14)$$

As a result, using the filter design equations given in (11), (12), (13) and (14) a Cosh window can be designed to satisfy the prescribed filter characteristic given in terms of A_s and Δw.

For the sake of another comparison with Kaiser window, the far-end stopband attenuation and Figure 7 shows that as the transition width increases, the filters designed by Cosh

DOI: 01.IJCOM.02.03. 6
ACEEE

window performs better far-end suppression than the filters designed by Kaiser window.

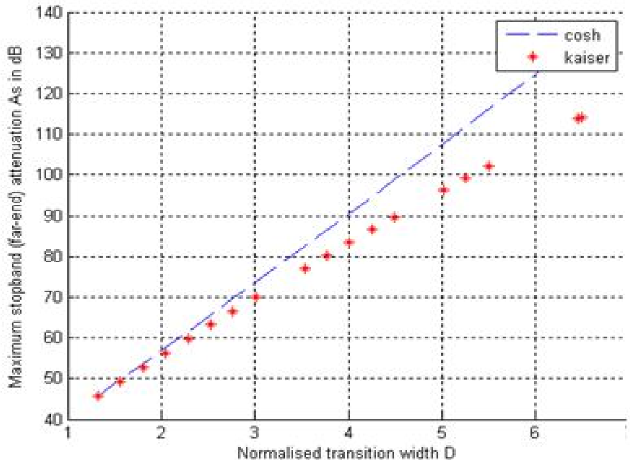

Figure 7. Comparison of the filters designed by Cosh and Kaiser windows in terms of the maximum stopband attenuation with N=127

V. Comparison Example and Simulation Results

Based on the findings of the previous section, a low pass nonrecursive filter that would satisfy the specifications (i) passband edge: ω_p (radian/sample) or f_p (hertz) (ii) stopband edge: ω_{st} (radian/sample) or f_{st} (hertz) (iii) passband ripple: A_p (iv) stopband ripple: A_s, (v) sampling frequency: ω_s *(radian/sample) or* F_s

Here an example is being presented which shows that the FIR Filter designed by Cosh window provides the better far-end stopband attenuation (maximum stopband attenuation) than the filter designed by the well known Kaiser window which is the figure of merit.

Example I: Design a lowpass FIR filter by Cosh window which meets the following specifications.

Sampling Frequency F_s=10 KHz., Passband Edge Frequency =1KHz.

Stopband Edge Frequency = 1.5KHz.

Passband attenuation = 36.5 dB.,

Stopband attenuation = 80 dB. and compare the results of FIR filter designed by Cosh window with the FIR filter designed by Kaiser window , simulation results are tabulated in TABLE I. From TABLE I, it is clear that filter designed by Cosh window provides better far-end stopband attenuation than filter designed by Kaiser window which is basically used for the application of sub-band coding and speech processing [7] and this is the greater advantage of filter designed by Cosh window than filter designed by Kaiser window. In TABLE I, FSA is far-end stopband attenuation.

The frequency response of filters designed by Kaiser and Cosh window are shown in Figure 8, and Figure 9.

Table I. Comparison Of Fir Filter Designed By Kaiser , Exponential And Cosh [9] Windows

S/No.	Parameters	Kaiser	Cosh
1	ω_c	0.7854	0.7854
2	D	6.2567	6.8247
3	Alpha (α)	9.912	10.152
4	N	126.5	140
5	FSA (in dB)	131.5	138.5

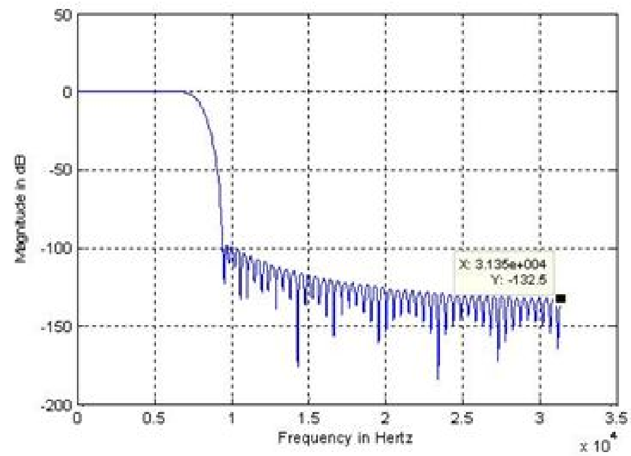

Figure 8. Frequency response of FIR Filter designed by Kaiser Window

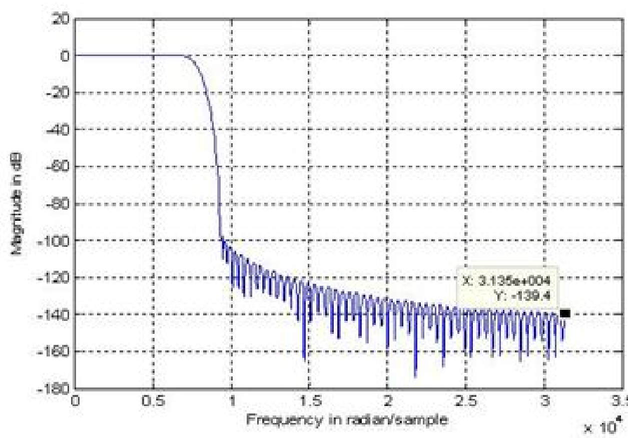

Figure 9. Frequency response of FIR Filter designed by Cosh window

VI. Conclusions

In this paper, the application of the proposed window in the nonrecursive digital FIR filter design is presented in which it is seen that the FIR filter designed by proposed window provides the worse minimum stopband attenuation but better far end attenuation than filter designed by well known Kaiser Window. The better far-end stopband attenuation in case of Cosh window shows the figure of merit and it is used for some applications such as sub band coding and speech processing. Simulation & experimental results showing a good agreement with theory has been provided

DOI: 01.IJCOM.02.03.6

REFERENCES

[1] A. Antoniou," Digital signal processing:"Signal, systems, and filters, McGraw-Hill, 2005.
[2] J. F. Kaiser and R. W. Schafer, "On the use of the Io-sinh window for spectrum analysis" IEEE Trans. Acoustics, Speech, and Signal Processing, vol.28, no.1, pp. 105-107, 1980.
[3] J. F. Kaiser, "Nonrecursive digital filter design using I0-sinh window function" in Proc. IEEE Int. Symp. Circuits and Systems (ISCAS'74), San Francisco, Calif, USA, pp.20-23, April 1974.
[4] C.L. Dolph, "A Current Distribution for Broadside Array, Which Optimizes the Relationship Between Beam width and Side-lobe Level" Proc. IRE, vol.34, pp.335-348, June 1946.
[5] T. Saramaki, "A class of window functions with nearly minimum sidelobe energy for designing FIR filters" in Proc. IEEE Int. Symp. Circuit and Systems (ISCAS'89), Portland, ORE, USA, vol.1, pp.359-362
[6] A. Jain, R. Saxena and S. C. Saxena, "A simple alias-free QMF system with near-perfect reconstruction" J. Indian Ins. Sci., Jan-Feb, no.12, pp. 1-10, 2005.
[7] K.Avci, A.Nacaroðlu, "A New Window Based on Exponential Function". Proceedings of 3rd International Conference on Information & Communication Technologies: from Theory to Applications (ICTTA'08), pp. 289-290, Damascus, Syria, April 2008.
[8] S. W. A. Bergen ,"Design of the Ultra spherical Window Function and Its Applications", University of Calgary, Tech. Rep. 2005
[9] F. J. Harris, "On the use of windows for harmonic analysis with the discrete Fourier transform," Proc. IEEE, vol. 66, pp. 5 1-83, Jan. 1978.

DOI: 01.IJCOM.02.03. 6